DISPATCHING SHIPMENTS AT MINIMAL COST WITH MULTIPLE MODE ALTERNATIVES

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Numerous organizations are involved in distributing goods and materials. When shipping alternatives exist, the selection of the appropriate shipping alternative (mode) for each shipment may result in significant cost savings. This paper describes a highly flexible dispatching system that selects the shipping mode to be used for each shipment while minimizing the overall shipping costs.

Multiple shipping modes must be considered when one faces economies of scale in shipping. On the one hand, combining shipments on a truck route usually results in reduced cost per shipment, but on the other hand, a common carrier alternative almost always exists. Therefore, nearly every major shipper faces shipping mode alternatives.

Generally, shipping by private (or dedicated) fleet is cheaper than other alternative modes, provided that the fleet is fully utilized. However, in order to assure full utilization, the variability in daily demand for transportation services compels operators to size their fleet below their long-term average transportation needs, and to supplement their fleet by outside (contract and/or common) carriers, as needed. The variability in demand for transportation services may stem from both internal and external sources, such as natural variability in customers' demand, which is manifested in different shipment (order) sizes, different sets of delivery/pick-up locations visited in a shift, promotions, and salesperson incentive structures. Even when an organization uses only private (or dedicated) fleet, but the fleet's truck physical characteristics and/or costs are not homogeneous, it faces

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a multiple mode situation. When multiple mode alternatives exist, minimizing miles, hours, or number of trucks used is deceiving. One has to minimize total shipping costs. The importance of cost minimization cannot be overstated, especially when we take into account anomalies of regulated rates.

The vast body of literature dealing with transportation routing and scheduling[1] is focused on homogeneous fleets and tries to minimize miles, hours, or number of trucks used. This focus stems from the recognition that transportation costs are neither linear nor continuous. Thus, incorporation of these costs into solvable mathematical models is hard (or even sometimes impossible), and one tends to resort to using proxy measures of effectiveness, such as miles, hours, or number of trucks used. Little published work deals with a multi-modal dispatching environment. Several works deal with the fleet size and mix problem when multiple types of trucks are available.[2] Two articles[3] deal with private fleet sizing in the presence of a common carrier alternative. Solutions to these problems set the stage for the daily dispatching environment, which is the topic of this paper. Ballou and Chowdhury and Pooley deal with modal choice in dispatching shipments, but only in a limited manner.[4] Both consider a choice between two modes only, and use heuristic procedures to solve the problem. In contrast, the system presented here accommodates any practical number of modes and gives the optimal (minimal cost) solution (dispatch).

At this point it is beneficial to clarify the terms used in this paper. A "shipment" is an "order" to be delivered or picked up from a location. The activity of delivery or pick up of an order is called a "task." A "route" is a sequence of locations to be visited by the truck between two consecutive visits to the source. A "schedule" is the work plan of a truck during one work shift. A schedule may consist of one or more routes (with reloads at the source) and includes the expected time of each activity on the routes. Later on a schedule is also called a "column." A "dispatch" is a plan how to ship the orders, which consists of a set of schedules for the available trucks, and the disposition of the orders that are not assigned to specific trucks. A "mode" is one type of transportation alternative that differs from other modes in its costs and/or its physical characteristics (i.e., different types of trucks are considered different modes).

The next section describes the operational environment for which the system was developed, followed by the mathematical model used to provide the optimal

solution. The various components of the dispatching system are then described in some detail, and finally, implementation experience is discussed.

OPERATIONAL ENVIRONMENT

A major U.S. corporation operates a dozen manufacturing plants in the continental United States. Each plant manufactures a common product line, which it distributes in its marketing region, and some specialty products (manufactured only by that plant), which it distributes nationwide. Adjacent to each plant is a mixing warehouse from which the products are shipped to retail outlets, industrial customers, and other plant warehouses. Thus, every plant ships both regionally and nationwide.

The products are packaged either in consumable or in reusable containers, which have to be returned to the plant. At each plant a mixed fleet of private (or dedicated) trucks is based, and that fleet is used to deliver shipments to customers, to pick up empty containers from customers, and to pick up packaging materials and materials for production from vendors. The various trucks may differ in their sizes, physical characteristics, equipment, compartments, operation rules, and cost structures (i.e., the same trip may cost differently on different trucks). The goods are shipped in box trailers, and the trucks are usually assigned multiple-stop trips. Shipment sizes vary from several pounds up to a truckload. Some of these trucks may perform more than one trip per day with reload at the plant. The private (or dedicated) fleet is usually kept within a one day radius of its plant, but in certain plants overnight trips are considered. Shipments farther away are assigned either to contract carriers (truckload or less-than-truckload), to a common carrier, or may be shipped to a pool point via linehaul, and then delivered locally by a dedicated carrier. Shipments within the operating radius of the private fleet may be carried by that fleet or assigned to a contract or common carrier. The private (dedicated) fleet operates one delivery shift per day. Due to the characteristics of the operation, there is very little overlap among the plants, and therefore each plant is dispatched separately. Table 1 presents the essential characteristics of this dispatching problem.

Customer service considerations dictate the latest shipping date for each order, but orders may be shipped early if it is found to be economically beneficial to do so. An order whose shipping date has arrived must be shipped by one mode or another. Table 2 lists the alternate modes that may be used to ship any given order.

TABLE 1

CHARACTERISTICS OF THE DISPATCHING PROBLEM

✔	Single Source
✔	Packaged Goods
✔	Multiple Shipping Modes
✔	Delivery and Pick-up/Backhauls
✔	Mixed Fleets
✔	Multiple Stops per Route
✔	Multiple Routes Per Truck Shift
✔	Objective: Shipping Cost Minimization

TABLE 2

ORDER SHIPPING ALTERNATIVES

1. Common Carrier (LTL)—The order is given to a common carrier (not consolidated with other orders) at a known cost.
2. Private Truck—The order is a stop on a route. The truck is paid by miles and hours, and returns to the source.
3. Dedicated Carrier Truck—The order is a stop on a route. The truck is paid by miles and hours (minimal charges apply) and returns to the source.
4. Truckload Carrier—The order is the final destination or a stop-off. The truck is paid per mile and the rate is determined by the final destination. Stop-off charges apply.
5. Via Pool Point—The order is consolidated in a line haul to a pool point (by method 2-4 above) and local delivery is performed by method 1-3 above.

Due to physical, contractual, and policy limitations, not every order is compatible with every type of truck (mode). The orders have to be assigned to compatible trucks in a manner that will not violate any of the limitations on the orders or the trucks and will minimize their shipping costs. We assign the orders to the trucks (shipping modes), and route the trucks at minimal cost, by using an Elastic Set Partitioning model (ESP), which is described in the next section.

MATHEMATICAL OPTIMIZATION MODEL

Three basic approaches exist for solving shipment dispatching problems (as well as other resource allocation problems): optimization, heuristics, and simulation. The relative merits of each one of these approaches can be found elsewhere.[5] We prefer to use optimization whenever possible because it assures the best solution; once we find that solution we know that no better solution exists. Even when the overall best solution is not found, we end up with a measure of the quality of the solution that we find. The optimization model we use to dispatch the shipments is an Elastic Set Partitioning model.

In a recent paper Pooley addressed the same type of problem that we face, but he used heuristic and allowed the consideration of only two modes. We allow multiple modes and (optimally) solve larger problems. Moreover, this work goes beyond the discussion of concepts and experiments and presents a system that operates daily in an industrial setting.

Elastic Set Partitioning is an integer programming model that is an extension of the familiar set partitioning model. Set Partitioning (SP) models have been used for scheduling transportation,[6] but authors usually had to resort to heuristics, which meant no assurance of optimal solutions, and often inability to measure the quality of the solution. The advantage of SP models is that they can accommodate nonlinear and discrete costs, mixed fleets, and a large variety of fleet operating rules. This is achieved by generating a large number (or all) of feasible (alternate) schedules for each truck and selecting one schedule for each truck in a manner that minimizes the cost of performing all the shipments. The major disadvantage of SP models is that they are hard to solve optimally.

In Elastic Set Partitioning we allow violation of the set partitioning constraints at a penalty, and the objective function minimizes the costs of the selected schedules plus the constraints violation penalties. ESP is a more compact formulation of the

problem because alternatives such as truck idleness, LTL, or common carrier shipments are comprehended through the violation penalties and do not have to be included as explicit constraints. The mathematical formulation of the ESP model is provided in the Appendix.

Table 3 presents data of a small simplified example with 2 trucks and 5 orders, and Table 4 presents the corresponding set partitioning model. Each column in the model consists of a feasible truck schedule, and the 1's stand for the specific truck and the tasks included in that schedule (e.g., schedule 5 shows truck 1 carrying orders 2 & 3). A binary variable represents each column, and the constraints assure that each truck is assigned exactly one schedule (column) and that each task is performed exactly once. The cost of each column is calculated based on the truck and orders involved in that specific schedule. In this example one feasible solution is to select columns 3 and 17 (at a cost of 431), another feasible solution consists of columns 9 and 13 (at a cost of 488), and a third one includes columns 10 & 15 (422). The optimal solution is the feasible solution that has the minimal total cost. This sample problem has only three feasible solutions, and they are summarized in Table 5. The optimal solution in this example is #3, which has the lowest total cost (but not the lowest milage!).

In ESP we allow violation of the equality constraints at a penalty, which is added to the cost of the specific solution. The selected solution is the one with the minimal total costs (violation penalties included). We solve the ESP problems on a mainframe or a microcomputer using our proprietary software.[7] General purpose software packages for linear or integer programming (e.g., LINDO, MPS III) are not designed to take advantage of the special structure of the ESP model, and therefore will require unacceptable solution times (several orders of magnitude larger than ours). Our solution time depends on the computer capabilities, the number of alternate schedules (columns) generated, the density of the columns (average number of tasks in a column), and the required optimality gap (an objective assessment of how close the schedule produced is to optimality). For smaller size problems we evaluate all feasible routes for each truck type, whereas for larger size problems we generate only good routes. The size of the problem that can be solved by our approach depends on multiple factors. As a general guideline, when all orders are compatible with all truck types, and trucks are assigned a single route per shift, the product of: (number of orders) x (number of truck types) x (maximal number of orders per truck route) should not exceed 20,000. However, often not all orders

TABLE 3

SMALL EXAMPLE DATA

ORDERS

Number	Size	Location (miles)* X	Location (miles)* Y
1	2	25	38
2	1	63	0
3	6	12	-25
4	12	-38	-12
5	4	-38	50

*The source is at (0,0).

TRUCKS

Number	Size	Maximum Stops	Cost ($/Mile)
1	10	3	1.00
2	20	3	1.50

Truck #2 doesn't have the equipment required to deliver order #1.

are compatible with all truck types (e.g., when private/dedicated trucks are limited to a certain radius from the source, orders beyond that radius must be assigned to a contract or common carrier), and then larger problems can be solved.

We have routinely solved large problems in several minutes on a fast 486 microcomputer to within 0.1% of optimality. The largest problem solved had 250 orders and 40 trucks, resulting in about 12,000 columns, and that problem took about half an hour to solve. We have also applied the same technique to dispatching multiple (truckload) trips per truck shift (with reloads at the source) and to dispatching ships. We are exploring the extension of this approach to multiple (alternate) order sourcing situations and to larger size problems.

TABLE 4

SET PARTITIONING MODEL FOR EXAMPLE PROBLEM

	Truck 1										Truck 2											
Schedule No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
Truck 1	1	1	1	1	1	1	1	1	1	1												= 1
Truck 2											1	1	1	1	1	1	1	1	1	1	1	= 1
Order 1	1	1	1						1	1												= 1
Order 2		1	1	1	1					1	1	1	1					1		1	1	= 1
Order 3			1		1	1	1					1	1	1	1						1	= 1
Order 4													1		1	1	1	1				= 1
Order 5							1	1	1	1							1	1	1	1	1	= 1
Miles	91	162	184	126	148	55	181	126	172	244	126	148	211	55	119	80	165	276	126	239	260	
Cost	91	162	184	126	148	55	181	126	172	244	189	222	316	82	178	120	247	414	189	358	390	

TABLE 5

SOLUTIONS OF SAMPLE PROBLEM

Feasible Solution Number	Columns	Total Miles	Total Cost
1	3 & 17	349	431
2	9 & 13	383	488
3	10 & 15	363	422

SHIPMENTS DISPATCHING SYSTEM

A useful dispatching system requires more than a mathematical model. It must facilitate communication between the user and the model. Therefore the ESP model is imbedded in a shipments dispatching system consisting of the following components: input data interface, data reader, column (schedules) generator, cost calculator, ESP solver, and output user interface.

The input data interface is used to provide the system with the appropriate data for the dispatching problem to be solved. These data include run parameters specifying the conditions of the specific run (including dispatching policy guidelines), information concerning the trucks, the shipments to be performed, carrier rates, the various locations involved, and geographical information (distances, speeds, travel times). Also, additional application specific information may be provided. For example, each order has a latest shipping date, which may be later than the day for which dispatching is done. Orders that may be shipped later are shipped only if the products are available and it is economical to do so (this is achieved by adjusting their violation penalties in the ESP model). Thus, the orders for a given day are divided into two categories: those that must be shipped and those that may be shipped. In order not to "starve" the private trucks in future days, the user retains control of the parameter that determines the economic incentive to ship orders early. Most of the input data does not change very frequently. Only the set of orders to be shipped and trucks availability must be updated daily. The detailed data concerning the entities involved in the dispatch allow control of the results through the specification of the actual data and nondefault values.

The data reader reads the input data for the specific problem instance, checks the data for completeness, consistency and accuracy, flags errors, and issues warnings. The data reader includes error correction routines that allow recovery if some data are incorrect (the incorrect data are replaced by default values).

The column generator uses the input data to generate alternate schedules (columns) for each truck while adhering to the operating rules specified in the run parameters and in each truck's and order's data. First an order-truck compatibility matrix is constructed, reflecting the operating rules of the specific truck and the requirements of the order, such as:

- Operating radius of the truck (where applicable)
- State rule for the truck (e.g., delivers out of state only)
- Preferred carrier for the order (when specified and available)
- Truck's equipment (when specific equipment is required for delivery)
- Truck performs backhauls
- Truck's capacity (weight, cube)

Next, for each truck all compatible orders are selected and used to generate a large set of feasible schedules (columns). The generated schedules must satisfy all the limitations on the truck, such as:

- Number of stops per route
- Truck weight, volume, and floor space
- Route length (miles and hours)
- Pick-ups performed only if sufficient access is available for remaining deliveries

Schedules that do not satisfy all the requirements are rejected. The alternate schedules for each truck are generated by a modified sweep algorithm.[8] A ray is drawn from the source to a seed order location, and the ray is rotated clockwise

(or counter-clockwise). All order locations hit by the ray are added to the truck route until the truck capacity is exhausted, thus the truck route covers all orders (compatible with that truck) in a sector. In addition, all subsectors are also set-up as routes. Every order is used as a seed starting point for a sweep. (The schedules in Table 4 were generated in this manner.) The orders on each route are sequenced by a quadratic assignment algorithm. At this point each alternate schedule (column) is submitted to the cost calculator which calculates the cost of the schedule, and the problem is converted to a standard ESP format and submitted to the solver.

The ESP solver solves the specific ESP problem to within the specified optimality gap and provides one schedule for each truck, where the set of selected schedules minimizes the cost of performing the specific set of shipments.

The output user interface allows the user to review the proposed dispatch, the schedule of each truck, and the associated costs. This module permits the user to change the proposed solution (reassignment) and see the cost and service implications of such changes.

The objective of the system is not to replace the dispatcher, but rather to assist the dispatcher in decision making. Dispatching decisions are usually highly complex, and not all situations can be foreseen and built into a computerized system. The system facilitates manual preassignment of tasks to trucks (even when such a preassignment violates operational rules), leaving the optimization of the remaining tasks to the ESP model. Thus the dispatcher is in the driver's seat and can compare the results of intuition and experience with mathematically optimal solutions. Each one of the modules described above generates a detailed log that allows tracing what has happened, and why.

IMPLEMENTATION EXPERIENCE

The shipments dispatching system described above has been in daily use by a major U.S. corporation for more than a year for dispatching shipments from a dozen plants. The number of shipments per day in any given plant is up to 250 with up to 40 trucks to be dispatched. Optimal (minimal cost) solutions are provided to the user within a minute or two on a mainframe or a microcomputer.

The user organization has obtained the following economic benefits from the system:

- Cost savings of about one million U.S. dollars in the first year of implementation
- Fewer minimum charges
- Better utilization of truck capacity (more weight on the trucks)
- Early delivery of future orders (this was not expected)
- Unexpected routing of trucks (routes that would not have been considered by dispatchers)

In addition, several managerial benefits were derived:

- Reduced dependence on key personnel
- Uniform delivery policy
- Tighter management control on actual delivery

A dispatcher usually experiments with a given dispatch several times, first without imposing and limitations and then adding minor adjustments. For example, the system may assign too much work to a specific carrier. The dispatcher will then simply reduce the number of trucks available from that carrier and run the system again. Once the dispatcher is satisfied with the dispatch, it is submitted to the truck terminals for execution.

Two major problems were encountered during the implementation of the system, and both are related to the provision of input data. First, in order to evaluate alternate routes, distances between stops on the routes are necessary. These distances are derived from a geographical coding system. Each location is assigned to a geographical zone, and the distance between any two locations is calculated by the shortest route between their zones (with appropriate adjustments for intra-zone distances). Second, carrier rates had to be provided for common and contract carriers. This issue is critical, especially for multi-stop carrier routes (e.g., stopoffs). Since the corporation was developing a carrier rates database for other purposes, that database was tapped to provide us the carrier rates. However, the provided carrier

rates are being regularly reviewed by the dispatchers in order to verify their validity and accuracy.

The "orders" data are provided daily from the order taking system, and the "trucks" data are maintained separately by the user.

SUMMARY

We have described an optimization model, imbedded in a shipments dispatching system, that provides optimal (minimal cost) dispatches where alternate shipping modes are available. This system accommodates any realistic number of modes, gives optimal (minimal cost) solutions to problems several times larger than reported before, and works daily in an industrial setting.

We have used the same technical approach to dispatching liquid bulk products by truck and by ships, and to determine whether to enter private/dedicated fleet operations, with comparable success. At the current state-of-the-art of computing and mathematical modeling, one has to justify not using optimization tools for dispatching.

NOTES

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APPENDIX

The formulation of the ESP model is as follows:

Indices:

- $i = 1, \ldots, n$ trucks
- j - an alternate schedule
- $j \in J(i)$ schedules requiring truck i
- $k = 1, \ldots, m$ tasks to be performed (shipments)
- $j \in J(k)$ schedules performing task k.

Data:

- C_j - Cost of entire schedule j (a function of the truck and the tasks in that schedule)
- $\underline{d}_i$, $\bar{d}_i$ - lower and upper constraint violation penalties for truck i
- $\underline{s}_k$, $\bar{s}_k$ - lower and upper constraint violation penalties for task k.

Decision variables:

- y_j = 1 if schedule j is selected; 0 otherwise
- $\underline{\delta}_i$, $\bar{\delta}_i$ - elastic (constraint violation) variables for truck i
- $\underline{\sigma}_k$, $\bar{\sigma}_k$ - elastic (constraint violation) variables for task k.

Model formulation (ESP):

$$\textit{Minimize:} \ \sum_j C_j y_j + \sum_{i=1}^{n} (\underline{d}_i \, \underline{\delta}_i + \bar{d}_i \, \bar{\delta}_i) + \sum_{k=1}^{m} (\underline{s}_k \, \underline{\sigma}_k + \bar{s}_k \, \bar{\sigma}_k) \qquad (1)$$

Subject to:

$$\sum_{j \in J(i)} y_j + \underline{\delta}_i - \overline{\delta}_i = 1 \text{ ; } \textit{for each truck } i \qquad (2)$$

$$\sum_{j \in J(k)} y_j + \underline{\sigma}_k - \overline{\sigma}_k = 1 \text{ ; } \textit{for each task } k \qquad (3)$$

$$y_j \in \{0,1\} \text{ ; } \textit{for each schedule } j \qquad (4)$$

$$\underline{\delta}_i, \overline{\delta}_i \in \{0,1\} \text{ ; } \textit{for each truck } i \qquad (5)$$

$$\underline{\sigma}_k, \overline{\sigma}_k \in \{0,1\} \text{ ; } \textit{for each task } k. \qquad (6)$$

Constraints (2) seek one schedule for each truck, where a lower violation represents total idleness of the truck (at a penalty representing the idleness cost) and an upper violation incurs a high schedule disruption penalty. Constraints (3) seek to perform all tasks; a lower violation represents a common carrier shipment (where permissible, at the appropriate cost), and an upper violation results in a high disruption penalty. The disruption penalties are assigned large numerical values in order to prevent such disruptions from occurring.

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